Influence of Conical Entry Regions on Laminar Entrance Losses for Newtonian Fluids

S. E. KAYE and S. L. ROSEN

Department of Chemical Engineering
Carnegie-Mellon University, Pittsburgh, Pennsylvania 15213

When a fluid passes in laminar flow through an abrupt contraction from a large diameter tube to one of smaller diameter, the excess entrance pressure loss is described by the equation (Sylvester and Rosen 1970a and 1970b)

$$\frac{\Delta P_{\text{ent}}}{\rho V^2/2g_c} = K + \frac{K'}{N_{Re}} \tag{1}$$

For Newtonian fluids, both the Hagenbach K and Couette K' coefficients must be functions of the entrance geometry only. The experimentally determined dependence of K and K' on $\beta (= D^2/D_0^2)$, the area contraction ratio has been reported for abrupt (180° total included angle) contractions (Kaye and Rosen, 1971).

Intuition would seem to indicate that excess entrance pressure losses can be reduced by making the transition from large to small tube more gradual. This easing of the transition is common practice in the design of capillary viscometers, for example. This work was undertaken to determine experimentally whether or not Equation (1) describes the excess pressure losses in conical entrance regions, and if so, to establish the dependence of K and K' on the cone angle for the laminar flow of Newtonian fluids. The relevant geometry and the excess entrance loss are defined in Figure 1.

Surprisingly, the Newtonian problem has received little experimental attention, while the far more complex case of non-Newtonian fluids has been studied more extensively. In the latter case, the entrance loss coefficients must depend on fluid properties—both equilibrium viscous and elastic—as well as geometry. The flow of polymer melts (Schott, 1964) and solutions (Chen, 1972) through conical entrance regions has been investigated. Tanner (1966) suggested the use of conical flows to obtain estimates of parameters in non-Newtonian constitutive equations. Sutterby (1965b, 1966) pursued this with dilute polymer solutions.

Some theoretical work has been done on laminar, Newtonian flow in tapered tubes. Usually, a spherical coordinate system is adopted, with the origin at the projected apex of the cone. Analytic solutions are generally based on the assumption of radial flow only, which is a physical impossibility (Dryden et al., 1956). Gibson (1909) was the first to attack the problem. Harrison (1919) obtained a creeping flow solution for pressure distribution subject to the assumptions of radial flow and slight taper. Bond (1925) developed this further to obtain an expression for the entrance loss. Oka (1964) obtained a similar relation in a study of velocity, pressure, and stress distributions for creeping flow through tapered tubes. Sutterby (1964) derived an equation analogous to Bond's for fast (inviscid) radial flow through a slightly tapered tube. Ashino (1969) also obtained a creeping-flow solution for a conical inlet which agrees well with Weissberg's (1962) result ($K' = 43.5$) for an abrupt ($\alpha = 90°$) contraction.

Boles et al. (1970) studied experimentally the flow of a Newtonian oil through a conical contraction. They obtained good agreement with Weissberg (1962) at $\alpha = 90°$ and with Oka (1964) at large angles, but not at smaller α.

Sutterby (1965a) performed a finite-difference analysis of flow through a slightly tapered tube. His solution asymptotically approaches the analytic solutions for slow (Bond, 1925) and fast (Sutterby, 1964) flow. Interestingly enough, his results for intermediate Reynolds num-

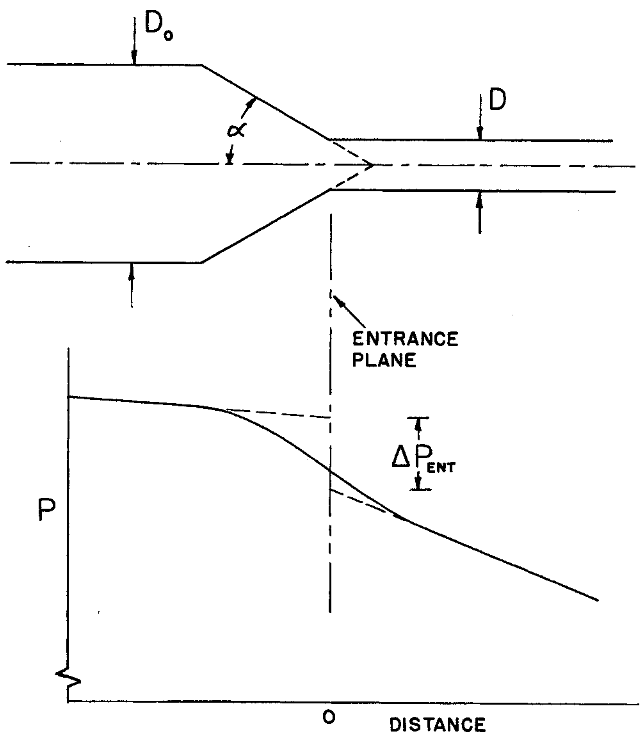

Fig. 1. Schematic of entrance geometry and pressure profile in the entrance region.

S. E. Kaye is with E. I. DuPont deNemours & Company, Martinsville, Virginia 24112.

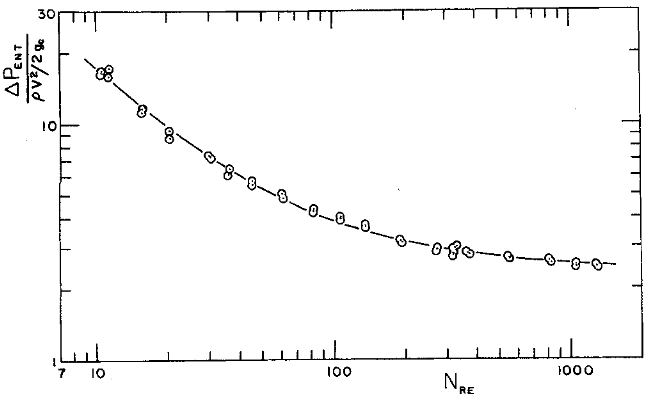

Fig. 2. Dimensionless entrance loss as a function of Reynolds number, $\alpha = 15°$, $\beta = 0.041$.

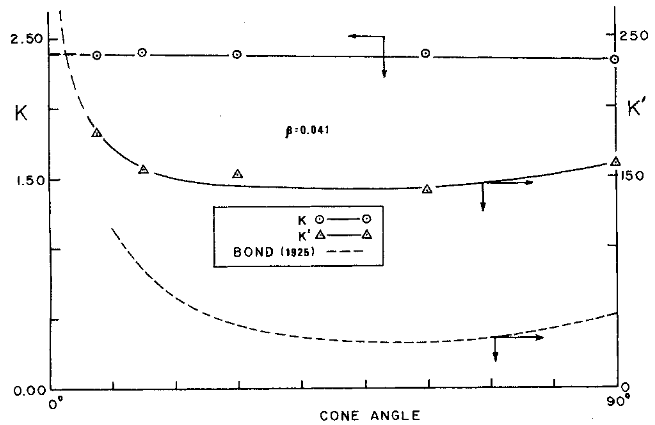

Fig. 3. Variation of entrance loss coefficients with cone angle, $\beta = 0.041$.

bers differ by up to 25% from the sum of the slow and fast flow solutions. This casts some doubt on the applicability of Equation (1), which is a sum of fast (K) and slow (K'/N_{Re}) flow solutions.

EXPERIMENT

The apparatus and procedures have been described previously (Kaye and Rosen, 1971). Cones with $\alpha = 60°$, 30°, 15°, and 7½° were machined from poly(methyl methacrylate) to fit in a 5.07 cm I.D. entrance (upstream) tube. With the test (downstream) section I.D. of 1.03 cm this gave a constant β of 0.041, well into the infinite contraction plateau previously established for abrupt contractions (Kaye and Rosen, 1971). Test fluids were aqueous glycerine solutions.

RESULTS

Figure 2 shows the data for $\alpha = 15°$. The curve drawn is Equation (1). This is typical of all the α's and shows quite clearly that Equation (1) does provide an accurate representation of experimental losses in conical entrance regions. (It should be noted that various definitions of Reynolds number have been adopted in work with conical flows. That used here is based on flow in the downstream tube, $N_{Re} = DV\rho/\mu$, consistent with our previous work.)

Figure 3 gives the dependence of K and K' on cone angle. The Hagenbach coefficient K is essentially independent of cone angle. This is not surprising, as at high Reynolds numbers, inertial effects predominate, and these should not be influenced by the nature of the contraction between upstream and downstream equilibrium flow regions. The Couette coefficient K' decreases slightly to a broad minimum between $\alpha = 30°$-$60°$, and begins to rise abruptly at small α. The experimental results for K' roughly parallel the analytic solution of Bond (1925) but are a factor of three higher. The analytic results of Ashino (1969) show a slight but monotonic increase in K' with decreasing α (12% between $\alpha = 90°$ and 45°), after which it shoots up rapidly below 45°.

This slight decrease in viscous losses caused by conical entrance regions is not totally unexpected. Observations of tiny bubbles in the system show clearly that the fluid forms its own cone of relatively stagnant fluid prior to an abrupt contraction. This effect has also been noted in the flow of polymer melts (Ballenger and White, 1971). The slight decrease is probably due to the elimination of the slow circulation which does occur in the relatively stagnant region. At small α, the cone effectively constricts the entrance region, increasing skin friction and therefore K'. As α goes to zero, the entrance region becomes infinitely long, and the viscous losses therein (as defined here) go to infinity.

CONCLUSIONS

Equation (1) adequately describes excess pressure losses for the laminar flow of Newtonian fluids through conical contractions. At low N_{Re} (< 10), tapering the entrance region can result in roughly a 10% decrease in entrance loss over an abrupt contraction, so in most cases, would hardly seem worth the effort.

NOTATION

α = entrance cone angle (1/2 total included angle)
β = area contraction ratio, $(D/D_0)^2$
D = diameter of test (downstream) tube
D_0 = diameter of entrance (upstream) tube
K = Hagenbach (fast-flow) coefficient, Equation (1)
K' = Couette (slow-flow) coefficient, Equation (1)
N_{Re} = Reynolds number in test section
V = volume average velocity in test section
ρ = fluid density
ΔP_{ent} = entrance pressure loss

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Effect of Geometric Parameters on the Friction Factor in Periodically Constricted Tubes

F. A. L. DULLIEN and M. I. S. AZZAM

Department of Chemical Engineering
University of Waterloo, Waterloo, Ontario, Canada

In a valuable contribution Payatakes et al. (1973) have solved the Navier-Stokes equations without neglecting any term, for a special case of periodically constricted tubes. Examples of their results have been given in two dimensionless diagrams (their Figures 8 and 9) which have been included here for reference as Figures 1 and 2. In both figures the friction factor of a uniform periodically constricted tube is plotted versus the Reynolds number. In Figure 1, the ratio $d_v^* = d_v/\tau$, where d_v is volumetric diameter and τ is length of a segment of a periodically constricted tube, was kept constant, equal to one; $\Delta r^* = (r_2 - r_1)/\tau$, where r_2 and r_1 are the maximum radius and the radius of the entrance constriction, respectively, was assigned different values, ranging from 0.5 to 0. In Figure 2, Δr^* was kept constant at 0.3, and d_v^* was assigned different values, ranging from 0.75 to ∞.

Payatakes et al. have stated that the conclusion reached by Batra (1969) and also reiterated by Batra, Fulford, and Dullien (1970), and by Dullien and Batra (1970), namely, that "The value of the friction factor increases with decreasing wave length to diameter ratio (< 0.5) by as much as 120% of the uniform tube value (wave amplitude to diameter ratios were in the range 0.13 to 0.25)" is incorrect. The criticism was based on an analysis of the above conclusion using the plots of Figure 2, notwithstanding the substantial difference between the geometry of the extensible flex-tubes used by Batra and that of their model. They also ignored the fact that Figure 2 does not apply to Batra's experiments in which Δr^* was not constant, but varied by as much as 27%. The reason for this variation is that it is not possible to vary $r_2 - r_1$ independently of τ in the case of extensible flex-tubes. Payatakes et al. were aware of these facts since they wrote "in the case of a flexible tube a decrease in τ/d_v is accompanied by a simultaneous increase in Δr^*".

Payatakes et al. allege also that in Batra's work "any change in the friction factor was attributed solely to the change in the ratio τ/d_v". It is unfortunate that these authors seem to have overlooked in their literature search the paper by Batra, Fulford, and Dullien (1970) where it has been demonstrated that in Batra's work the effect of τ/d_v on f_v far outweighed the effect of $(r_2 - r_1)/d_v$. This may be seen from Figures 3 and 4 with the accompanying tables, showing some of Batra's results. (We found it more convenient to use the dimensionless ratio $(r_2 - r_1)/d_v$ instead of $(r_2 - r_1)/\tau$.)

The data shows that both τ/d_v and $(r_2 - r_1)/d_v$ decreased whereas f_v increased. For obvious reasons a decrease in $(r_2 - r_1)/d_v$, that is, decrease in $(r_2 - r_1)$ for a fixed d_v, must cause a decrease in f_v, so that only the decrease in τ/d_v could have caused the observed increase in f_v.

The fact that decreasing τ for fixed values of d_v and $r_2 - r_1$, can indeed be expected to cause an increase in f_v may be understood if one first considers the limiting case of a uniform tube (one, without any constrictions) for which $\tau/d_v = \infty$, and then starts to introduce periodic irregularities into the tube walls to produce a culvert-like surface. It is obvious that, for a fixed d_v and $r_2 - r_1$, the friction factor f_v will tend to increase with decreasing wave length τ.

Hence, if f_v is regarded as a function of two variables

$$f_v = f_v[(r_2 - r_1)/d_v, \tau/d_v],$$

there can be little doubt that

$$\{\partial f_v/\partial[(r_2 - r_1)/d_v]\}_{\tau/d_v} > 0,$$

and

$$\{\partial f_v/\partial(\tau/d_v)\}_{(r_2-r_1)/d_v} < 0.$$

The first of these trends is also confirmed by the calculations of Payatakes et al. In Figure 1 $d_v^* = d_v/\tau = 1 =$ constant, implying that τ is constant for a fixed d_v.